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Non-minimal coupling to a Lorentz-violating background and topological implications

H. Belich[1,2,a], T. Costa-Soares[2,3,4,b], M.M. Ferreira Jr.[2,5,c], J.A. Helayël-Neto[2,3,d]

[1] Universidade Federal do Espírito Santo (UFES), Departamento de Física e Química, Av. Fernando Ferrari, S/N Goiabeiras, Vitória – ES, 29060-900, Brasil
[2] Grupo de Física Teórica José Leite Lopes, C.P. 91933, CEP 25685-970, Petrópolis, RJ, Brasil
[3] CCP, CBPF, Rua Xavier Sigaud, 150, CEP 22290-180, Rio de Janeiro, RJ, Brasil
[4] Universidade Federal de Juiz de Fora (UFJF), Colégio Técnico Universitário, av. Bernardo Mascarenhas, 1283, Bairro Fábrica – Juiz de Fora – MG, 36080-001, Brasil
[5] Universidade Federal do Maranhão (UFMA), Departamento de Física, Campus Universitário do Bacanga, São Luiz - MA, 65085-580, Brasil

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Abstract. The non-minimal coupling of fermions to a background responsible for the breaking of Lorentz symmetry is introduced in Dirac's equation; the non-relativistic regime is contemplated, and the Pauli equation is used to show how an Aharonov–Casher phase may appear as a natural consequence of the Lorentz violation, once the particle is placed in a region where there is an electric field. Different ways of implementing the Lorentz breaking are presented and, in each case, we show how to relate the Aharonov–Casher phase to the particular components of the background vector or tensor that realizes the violation of Lorentz symmetry.

1 Introduction

In the beginning of the nineties, Carroll, Field and Jackiw [1] have considered a Chern–Simons-like odd-CPT term able to induce the violation of Lorentz symmetry in $(1+3)$ dimensions. In a more recent context, some authors [2] have explored the possibility of Lorentz symmetry breaking in connection with string theories. Models with Lorentz and CPT breakings were also used as a low-energy limit of an extension of the standard model, valid at the Planck scale [3]. In this case, an effective action is obtained that incorporates CPT and Lorentz violation and keeps unaffected the $SU(3)\times SU(2)\times U(1)$ gauge structure of the underlying theory. This fact is of relevance in that it indicates that the effective model may preserve some properties of the original theory, like causality and stability. Though of Lorentz symmetry is closely connected to stability and causality in modern field theories, the existence of a causal and unitary model with violation of Lorentz symmetry is in principle possible and meaningful on physical grounds.

In the latest years, Lorentz-violating theories have been investigated under diverse perspectives. In the context of $N=1$ supersymmetric models, there have appeared two proposals: one which violates the algebra of supersymmetry (first addressed by Berger and Kostelecky [4]), and another that preserves the (SUSY) algebra and yields the Carroll–Field–Jackiw model by integrating on Grassmann variables [5]. The study of radiative corrections arising from the axial coupling of charged fermions to a constant vector has raised a controversy on the possible generation of the Chern–Simons-like term that has motivated a great deal of work in the literature [14]. The rich phenomenology of fundamental particles has also been considered as a natural scenario for the search of indications of the breaking of these symmetries [8,9], imposing stringent limitations on the factors associated with such a violation. The traditional discussion concerning space-time varying coupling constants has also been addressed in the light of Lorentz-violating theories [10], with interesting connections with the construction of supergravity. Moreover, measurements of radio emission from distant galaxies and quasars put in evidence that the polarization vectors of the radiation emitted are not randomly oriented as naturally expected. This peculiar phenomenon suggests that the space-time intervening between the source and the observer may be exhibiting some sort of optical activity (birefringence), whose origin is unknown [11]. Different proposals for implementation of Lorentz violation have been recently considered; one of them consists of ob-

[a] e-mail: belich@cce.ufes.br
[b] e-mail: tcsoares@cbpf.br
[c] e-mail: manojr@ufma.br
[d] e-mail: helayel@cbpf.br

taining it from the spontaneous symmetry breaking of a matter vector field [12].

Our approach to the Lorentz breaking consists of adopting the 4-dimensional version of a Chern–Simons topological term, namely $\epsilon_{\mu\nu\kappa\lambda}v^{\mu}A^{\nu}F^{\kappa\lambda}$ (also known as the Carroll–Field–Jackiw term [1]), where $\epsilon_{\mu\nu\kappa\lambda}$ is the 4-dimensional Levi-Civita symbol and v^{μ} is a fixed 4-vector acting as a background. A study of the consequences of such breaking in QED has been extensively analyzed in the literature [13,14]. An extension of the Carroll–Field–Jackiw (CFJ) model to include a scalar sector that yields spontaneous symmetry breaking (Higgs sector) was recently developed and analyzed, resulting into an Abelian–Higgs CFJ electrodynamics (AHCFJ model) with violation of Lorentz symmetry [15]. Afterwards, the dimensional reduction of the CFJ and AHCFJ models (to $1+2$ dimensions) were successfully carried out in [16,18], respectively, yielding a planar Maxwell–Chern–Simons and a Maxwell–Chern–Simons–Proca electrodynamics mixed to a scalar field (responsible for the Lorentz violation). It should be here stressed that these planar models do not present the causality and unitarity problems that affect the original CFJ and AHCFJ models; instead, they set out as entirely consistent planar theories, whose properties have recently been investigated under different aspects [19,20].

Topological effects in quantum mechanics are phenomena that present no classical counterparts, being associated with physical systems defined on a multiply connected space-time. Specifically, considering a charged particle that propagates in a region with external magnetic field (free force region), it is verified that the corresponding wave function may develop a quantum phase: $\langle b|a\rangle_{\mathrm{in}\,A} = \langle b|a\rangle_{A=0}.\left\{\exp\left(\mathrm{i}q\int_a^b \mathbf{A}.\mathrm{d}\mathbf{l}\right)\right\}$, which describes the real behavior of the electron propagation. This issue has received considerable attention since the pioneering work by Aharonov and Bohm [21], where they demonstrate that the vector potential may induce physically measurable quantum phases even in a field-free region, which constitute the essence of a topological effect. The induced phase does not depend on the specific path described by the particle, neither on its velocity (non-dispersiveness). Instead, it is intrinsically related to the non-simply connected nature of the space-time and to the associated winding number. Many years later, Aharonov and Casher [22] argued that a quantum phase also appears in the wave function of a neutral spin-$\frac{1}{2}$ particle with anomalous magnetic moment, μ, subject to an electric field arising from a charged wire. This is the well-known Aharonov–Casher (A-C) effect, which is related to the A-B effect by a sort of duality transformation.

This effect can be investigated by taking into account the non-relativistic limit of the Dirac's equation [23] with the Pauli-type non-minimal coupling. Concerning these phase effects, other developments over the past years have raised a number of interesting questions; in connection to the latter, locality and topology are being invoked in a more recent context [24]. The local or topological nature of the generated phase can change according to each situation, as in the case of the A-B effect in molecular systems, which is neither local nor topological, being closer to the A-C effect. For instance, the work of [25] discusses the A-C phase in a planar model in order to demonstrate that this effect is essentially non-local in the context of a non-relativistic superconductor. The formal correspondence between the A-B and A-C phases at a microscopic level, as long as their topological nature is concerned, is considered in [26]. In the context of ultra-cold atoms, it was shown that the vortex model of Bose–Einstein condensates is described by a Lagrangian with an A-C extra term [27].

In this work, we focus on the investigation of non-minimal coupling terms in the context of Lorentz-violating models involving some fixed background and the gauge and fermion fields. The main purpose is to figure out whether such new couplings are able to induce the A-C effect. In this sense, we follow a single procedure: writing the spinor field in terms of its small and large components, we arrive at the Pauli equation (once the non-relativistic limit of the Dirac equation is considered) and identify the generalized canonical momentum, which in this approach plays a central role for the determination of the induced quantum phases.

This paper is organized as follows. In Sect. 2, several kinds of Lorentz-violating non-minimal couplings are analyzed in connection with the possibility of generating an A-C quantum phase. Initially, we consider the presence of the non-minimal term, $\mathrm{i}gv^{\nu}\widetilde{F}_{\mu\nu}$, in the covariant derivative, to account for the coupling of a neutral test particle to the Lorentz breaking background. In the sequel, working out the non-relativistic limit, we derive the Pauli equation and write down the generalized canonical momentum, whose composition may indicate the appearance of an A-C phase. In this case, the background 3-vector plays the role of the magnetic moment of the neutral particle. As a second case, we regard a non-minimally torsion-like (γ_5-type) coupling with the Lorentz-violating background in the context of the Dirac equation. No A-C phase is generated in this case. In another situation, a background tensor ($T_{\mu\nu}$) responsible for the violation of Lorentz symmetry is non-minimally coupled to the electromagnetic and Dirac fields. It is observed that the anti-symmetric part of this tensor induces an A-C phase. As a final investigation, we simultaneously considered the non-minimal Lorentz breaking coupling and the (Pauli) standard non-minimal coupling in order to study the competition between these terms for the generation of an A-C phase. It has been then verified that only the standard Pauli magnetic coupling yields an A-C effect. Our final discussion is presented in Sect. 3.

2 Lorentz-violating non-minimal couplings, Pauli equation and the Aharonov–Casher phase

2.1 Non-minimal coupling to the gauge field and background

The first case to be analyzed starts with the gauge invariant Dirac equation,

$$(\mathrm{i}\gamma^{\mu}D_{\mu}-m)\Psi=0, \tag{1}$$

where the covariant derivative with non-minimal coupling is chosen to be

$$D_{\mu}=\partial_{\mu}+\mathrm{i}eA_{\mu}+\mathrm{i}gv^{\nu}\widetilde{F}_{\mu\nu}, \tag{2}$$

whereas v^{μ} is a fixed 4-vector acting as the background which breaks the Lorentz symmetry [1]. The explicit representation of the γ-matrices used throughout is listed below:

$$\gamma^{0}=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},\ \overrightarrow{\gamma}=\begin{pmatrix}0 & \overrightarrow{\sigma}\\ -\overrightarrow{\sigma} & 0\end{pmatrix},\quad \gamma_{5}=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \tag{3}$$

where $\overrightarrow{\sigma}=(\sigma_{x},\sigma_{y},\sigma_{z})$ are the Pauli matrices. In order to simplify the calculations, the spinor Ψ should be written in terms of small (χ) and large (ϕ) 2-spinors, $\Psi=\begin{pmatrix}\phi\\ \chi\end{pmatrix}$, so that (1) splits into two equations for ϕ and χ:

$$\left(E-e\varphi-g\overrightarrow{v}\cdot\overrightarrow{B}\right)\phi-\overrightarrow{\sigma}\cdot(\overrightarrow{p}-e\overrightarrow{A}+gv^{0}\overrightarrow{B}-g\overrightarrow{v}\times\overrightarrow{E})\chi=m\phi, \tag{4}$$

$$-\left(E-e\varphi-g\overrightarrow{v}\cdot\overrightarrow{B}\right)\chi-\overrightarrow{\sigma}\cdot(\overrightarrow{p}-e\overrightarrow{A}+gv^{0}\overrightarrow{B}-g\overrightarrow{v}\times\overrightarrow{E})\phi=m\chi. \tag{5}$$

Writing the small component in terms of the large one (in the non-relativistic limit), one has

$$\chi=\frac{1}{2m}\overrightarrow{\sigma}\cdot\left(\overrightarrow{p}-e\overrightarrow{A}+gv^{0}\overrightarrow{B}-g\overrightarrow{v}\times\overrightarrow{E}\right)\phi. \tag{6}$$

Replacing such a relation in (5), one achieves the associated Pauli equation for the large component ϕ, namely

$$\left(E-e\varphi-g\overrightarrow{v}\cdot\overrightarrow{B}\right)\phi-\frac{1}{2m}\left(\overrightarrow{\sigma}\cdot\overrightarrow{\Pi}\right)\left(\overrightarrow{\sigma}\cdot\overrightarrow{\Pi}\right)\phi=m\phi, \tag{7}$$

where the canonical generalized moment is defined as

$$\overrightarrow{\Pi}=\left(\overrightarrow{p}-e\overrightarrow{A}+gv^{0}\overrightarrow{B}-g\overrightarrow{v}\times\overrightarrow{E}\right). \tag{8}$$

The presence of the term $g\overrightarrow{v}\times\overrightarrow{E}$, which possesses a non-vanishing rotation, is the aspect that determines the induction of the Aharonov–Casher effect. Indeed, the 3-vector background plays the role of a sort of magnetic dipole moment ($\overrightarrow{\mu}=g\overrightarrow{v}$) that gives rise to the A-C phase associated with the wave function of a neutral test particle ($e=0$), for which the Aharonov–Bohm effect is absent. In the case of a charged particle, the non-minimal coupling of (2) brings about simultaneously the A-B and A-C phases. For a neutral particle under the action of an external electric field, the A-C phase induced as a consequence of the Lorentz symmetry violation read $\Phi_{\mathrm{AC}}=\oint_{C}(g\overrightarrow{v}\times\overrightarrow{E})\cdot\mathrm{d}\overrightarrow{l}$ where C is a closed path.

With this result, we can comment on another possible non-minimal coupling, which has not been included in the covariant derivative (2), $\mathrm{i}hv^{\nu}F_{\mu\nu}$, with h being the coupling constant. It does not yield an A-C phase, but it rather implies an extra phase involving the magnetic field and it takes the form $\overrightarrow{v}\times\overrightarrow{B}$.

To write the Hamiltonian associated with the Pauli equation exhibited above, one should use the well-known identity

$$\left(\overrightarrow{\sigma}\cdot\overrightarrow{\Pi}\right)^{2}=\overrightarrow{\Pi}^{2}+\mathrm{i}\overrightarrow{\sigma}\cdot\left(\overrightarrow{\Pi}\times\overrightarrow{\Pi}\right), \tag{9}$$

which after some algebraic manipulations leads to

$$H=\frac{1}{2m}\overrightarrow{\Pi}^{2}+e\varphi-\frac{e}{2m}\overrightarrow{\sigma}\cdot(\overrightarrow{\nabla}\overrightarrow{A})+\frac{1}{2m}gv^{0}\overrightarrow{\sigma}\cdot(\overrightarrow{\nabla}\overrightarrow{B})+\frac{g}{2m}\overrightarrow{\sigma}\cdot\overrightarrow{\nabla}(\overrightarrow{v}\overrightarrow{E}), \tag{10}$$

where $\varphi\equiv A^{0}$.

We would like here to point out the work of [28], where the authors present a method for deriving the non-relativistic limit of free massive fermions from a rather general Lorentz-violating Lagrangian, by using the Foldy–Wouthuysen expansion. The result we have shown above, and the ones we shall present in the forthcoming cases, agree with the non-relativistic limits worked out in [28], by suitably identifying the combination $eA_{\mu}+\mathrm{i}gv^{\nu}\widetilde{F}_{\mu\nu}$ with the parameter a_{μ} in [28].

2.2 Torsion non-minimal coupling with Lorentz violation

In this section, one deals again with (1), now considering another kind of non-minimal coupling,

$$D_{\mu}=\partial_{\mu}+eA_{\mu}+\mathrm{i}g_{a}\gamma_{5}v^{\nu}\widetilde{F}_{\mu\nu}, \tag{11}$$

where the Lorentz-violating background, v^{μ}, appears coupled to the gauge field by means of a torsion-like term of chiral character.

Writing the spinor Ψ in terms of the so-called small and large components in much the same way as in the latter section, there appear two coupled equations for the 2-component spinors ϕ,χ,

$$\begin{aligned}&\left[(E-e\varphi)+\overrightarrow{\sigma}\cdot\left(g_a v^0\overrightarrow{B}-g_a\overrightarrow{v}\times\overrightarrow{E}\right)\right]\phi\\&-[\overrightarrow{\sigma}\cdot\left(\overrightarrow{p}-e\overrightarrow{A}\right)+g_a\overrightarrow{v}\cdot\overrightarrow{B}]\chi\\&=m\phi, \qquad (12)\\&-\left[(E-e\varphi)+\overrightarrow{\sigma}\cdot\left(g_a v^0\overrightarrow{B}-g_a\overrightarrow{v}\times\overrightarrow{E}\right)\right]\chi\\&+[\overrightarrow{\sigma}\cdot\left(\overrightarrow{p}-e\overrightarrow{A}\right)+g_a\overrightarrow{v}\cdot\overrightarrow{B}]\phi\\&=m\chi, \qquad (13)\end{aligned}$$

from which we can read the small component in terms of the large one:

$$\chi=\frac{1}{2m}\left[\overrightarrow{\sigma}\cdot\left(\overrightarrow{p}-e\overrightarrow{A}\right)+g_a\overrightarrow{v}\cdot\overrightarrow{B}\right]\phi. \qquad (14)$$

From (13) and (14), one obtains the corresponding Pauli equation,

$$\begin{aligned}&\left(E-e\varphi+\overrightarrow{\sigma}\cdot\left(g_a v^0\overrightarrow{B}-g_a\overrightarrow{v}\times\overrightarrow{E}\right)\right)\phi\\&-\left[\overrightarrow{\sigma}\cdot\left(\overrightarrow{p}-e\overrightarrow{A}\right)+g_a\overrightarrow{v}\cdot\overrightarrow{B}\right]\\&\quad\times\frac{1}{2m}\left[\overrightarrow{\sigma}\cdot\left(\overrightarrow{p}-e\overrightarrow{A}\right)+g_a\overrightarrow{v}\cdot\overrightarrow{B}\right]\phi\\&=m\phi, \qquad (15)\end{aligned}$$

whose structure reveals the canonical generalized moment by the usual relation, $\overrightarrow{\Pi}=\left(\overrightarrow{p}-e\overrightarrow{A}\right)$. Here, one notices that the non-minimal coupling gives rise only to an energy contribution denoted by H_{nm} and given by

$$\begin{aligned}H_{nm}=&\overrightarrow{\sigma}\cdot\left(g_a v^0\overrightarrow{B}-g_a\overrightarrow{v}\times\overrightarrow{E}\right)+\frac{1}{2m}\left(g_a\overrightarrow{v}\cdot\overrightarrow{B}\right)^2\\&+\frac{g_a}{2m}\overrightarrow{\sigma}\cdot\left(\overrightarrow{p}-e\overrightarrow{A}\right)\overrightarrow{v}\cdot\overrightarrow{B}\\&+\frac{g_a}{2m}\overrightarrow{v}\cdot\overrightarrow{B}\overrightarrow{\sigma}\cdot\left(\overrightarrow{p}-e\overrightarrow{A}\right), \qquad (16)\end{aligned}$$

so that the non-relativistic Hamiltonian becomes $H=\frac{1}{2m}\overrightarrow{\Pi}^2+e\varphi-\frac{e}{2m}\overrightarrow{\sigma}\cdot\nabla\times\overrightarrow{A}+H_{nm}$.

We can thus conclude that, if the fixed background is associated with the vector component of the torsion, as done in the work of [30], no A-C phase is induced. The coupling to the torsion contributes to the interaction energy, but contrary to the case contemplated in the previous section, the γ_5-type non-minimal coupling does not bring about any A-C phase.

2.3 Lorentz-violating non-minimal coupling to a tensor background

The starting point now is an extended Dirac equation minimally coupled to electromagnetic field, explicitly given by

$$(\mathrm{i}\gamma^\mu D_\mu-m+\mathrm{i}\lambda_1 T_{\mu\nu}\Sigma^{\mu\nu}+\mathrm{i}\lambda_2 T_{\mu\kappa}F^{\kappa}{}_{\nu}\Sigma^{\mu\nu})\Psi=0, \qquad (17)$$

where the covariant derivative is the usual one, $D_\mu=\partial_\mu+eA_\mu$, and the bilinear term, $\Sigma^{\mu\nu}=\mathrm{i}[\gamma^\mu,\gamma^\nu]/2$, is written as

$$\Sigma^{0i}=\mathrm{i}\begin{pmatrix}0 & \overrightarrow{\sigma}\\ -\overrightarrow{\sigma} & 0\end{pmatrix},\ \Sigma^{ij}=\varepsilon_{ijk}\sigma^k\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}.$$

Notice that in this case, the skew-symmetric tensor $T_{\mu\nu}$ is the element responsible for the Lorentz violation at the level of the fermionic coupling. In analogy to what occurs when fermion couplings violate Lorentz symmetry, by means of a term of the type $b_\mu\overline{\Psi}\gamma^\mu\gamma_5\Psi$ [31], we here propose Lorentz violation by taking into account fermionic couplings in the form $\overline{\Psi}\Sigma^{\mu\nu}\Psi T_{\mu\nu}$ and $\overline{\Psi}\Sigma^{\mu\nu}\Psi F_{\mu\kappa}T^{\kappa}{}_{\nu}$.

Following the same procedure as previously adopted, we write down 2-component equations:

$$\begin{aligned}&(E-e\varphi)\,\phi-\vec{\sigma}.\left(\overrightarrow{p}-e\overrightarrow{A}\right)\chi+4\mathrm{i}\lambda_1 T_{0i}\sigma^i\chi\\&+\lambda_1 T_{ij}\varepsilon_{ijk}\sigma^k\phi+2\mathrm{i}\lambda_2 T^{0i}F_{ij}\sigma^j\chi\\&+T^{i0}F_{0k}\varepsilon_{ijk}\sigma^j\phi+\lambda_2 T^{ij}F_{jk}\varepsilon_{ijk}\sigma^j\phi+2\mathrm{i}\lambda_2 T^{ij}F_{j0}\sigma^i\chi\\&=m\phi, \qquad (18)\\&(E-e\varphi)\,\chi+\vec{\sigma}.\left(\overrightarrow{p}-e\overrightarrow{A}\right)\phi+4\mathrm{i}\lambda_1 T_{0i}\sigma^i\phi\\&+\lambda_1 T_{ij}\varepsilon_{ijk}\sigma^k\chi+2\mathrm{i}\lambda_2 T^{0i}F_{ij}\sigma^j\phi\\&+T^{i0}F_{0k}\varepsilon_{ijk}\sigma^j\chi+\lambda_2 T^{ij}F_{jk}\varepsilon_{ijk}\sigma^j\chi+2\mathrm{i}\lambda_2 T^{ij}F_{j0}\sigma^i\phi\\&=m\chi. \qquad (19)\end{aligned}$$

The pair of the 2-component equations above involves both the external electric and magnetic fields. As the A-C effect is the main subject of interest of this investigation, we shall consider a vanishing magnetic field, $F_{ij}=0$, which allows one to achieve the following expression relating small and large components:

$$\begin{aligned}\chi=\frac{1}{2m}\Big[&\overrightarrow{\sigma}.\left(\overrightarrow{p}-e\overrightarrow{A}\right)+\lambda_1 4\mathrm{i}T_{oi}\sigma^i\\&+\lambda_2 2\mathrm{i}(\overrightarrow{T}\times\overrightarrow{E})_i\sigma^i\Big]\phi, \qquad (20)\end{aligned}$$

where we used $T^{ij}F_{j0}=(\overrightarrow{T}\times\overrightarrow{E})_i$. By factoring out the Pauli matrices, $\overrightarrow{\sigma}$, the following canonical moment can be identified:

$$\overrightarrow{\Pi}=\left(\overrightarrow{p}-e\overrightarrow{A}-4\lambda_1\overrightarrow{T}_1-2\lambda_2\overrightarrow{T}_2\times\overrightarrow{E}\right), \qquad (21)$$

where we have distinguished the "electric" and the "magnetic" components of the tensor, respectively defined by $T_{0i}=\overrightarrow{T}_1$, $T^{ij}=\overrightarrow{T}_2$.

Replacing this in (20), we get the following equation for the large spinor component:

$$\begin{aligned}&\left(E-e\varphi-\lambda_1 T_{kj}\varepsilon_{ijk}\sigma^i+\lambda_2 T^{j0}F_{0k}\varepsilon_{ijk}\sigma^i\right)\phi\\&-\frac{1}{2m}(\overrightarrow{\sigma}\cdot\overrightarrow{\Pi})(\overrightarrow{\sigma}\cdot\overrightarrow{\Pi})\phi=m\phi. \qquad (22)\end{aligned}$$

Here again two kinds of quantum phases appear, one governed by λ_1 and the other by λ_2. However, having in mind that our purpose is to clarify how the A-C phase can emerge, we can take $\lambda_1 = 0$. The λ_2-term, with the componentof $T^{\mu\nu}$, gives rise to the A-C contribution. Should we take $\lambda_1 = 0$, the Lorentz breaking term would not impose $T_{\mu\nu}$ to be skew-symmetric. Indeed, if $T_{\mu\nu}$ were taken to be a general tensor, the conditions for the A-C to appear (no magnetic field but only an external electric field) would anyhow select the anti-symmetric magnetic component, $T_{ij} = -T_{ji}$; the A-C phase is therefore induced by the anti-symmetric piece of $T_{\mu\nu}$.

In this case, the Hamiltonian is given as follows:

$$H = \frac{1}{2m}\overrightarrow{\Pi}^2 + e\varphi - \frac{e}{2m}\overrightarrow{\sigma}\cdot\overrightarrow{\nabla}\times\overrightarrow{A} + \frac{1}{2m}\overrightarrow{\sigma}\cdot\overrightarrow{\nabla}\times\left(4\mathrm{i}\lambda_1\overrightarrow{T}_1 + 2\mathrm{i}\lambda_2\overrightarrow{T}_2\times\overrightarrow{E}\right). \tag{23}$$

2.4 Competition between Lorentz-preserving and Lorentz-violating non-minimal couplings

In this section, we would like to compare the specific non-minimal coupling with Lorentz violation exhibited in (1) and (2) with the standard non-minimal coupling that generates the usual Aharonov–Casher effect, in such a way as to verify how these terms are related to the development of an A-C phase.

The gauge invariant Dirac equation from which we shall compute the Pauli equation is

$$(\mathrm{i}\gamma^\mu D_\mu - m + f\Sigma^{\mu\nu}F_{\mu\nu})\Psi = 0, \tag{24}$$

where the covariant derivative with non-minimal coupling is the one given in (2). Following the same procedure already adopted, we shall work out the non-relativistic limit of the Dirac equation. Writing the spinor Ψ in small and large components, from (24) there result two equations:

$$\left[E - e\varphi - g\overrightarrow{v}\cdot\overrightarrow{B} + \left(2f\Sigma^{0i}F_{0i} + f\Sigma^{ij}F_{ij}\right)\right]\phi - \overrightarrow{\sigma}\cdot\left(\overrightarrow{p} - e\overrightarrow{A} + gv^0\overrightarrow{B} - g\overrightarrow{v}\times\overrightarrow{E}\right)\chi = m\phi, \tag{25}$$

$$\left[-(E - e\varphi - g\overrightarrow{v}\cdot\overrightarrow{B}) + \left(2f\Sigma^{0i}F_{0i} + f\Sigma^{ij}F_{ij}\right)\right]\chi + \overrightarrow{\sigma}\cdot\left(\overrightarrow{p} - e\overrightarrow{A} + gv^0\overrightarrow{B} - g\overrightarrow{v}\times\overrightarrow{E}\right)\phi = m\chi. \tag{26}$$

In the non-relativistic limit, there appears the following Pauli equation:

$$\left(E - e\varphi - g\overrightarrow{v}\cdot\overrightarrow{B} + f\varepsilon_{ijk}\sigma^k F_{ij}\right)\phi - \left(\overrightarrow{\sigma}\cdot\overrightarrow{\mathcal{P}}\right)\frac{1}{2m}\left(\overrightarrow{\sigma}\cdot\overrightarrow{\mathcal{P}}\right) = m\phi, \tag{27}$$

where $\overrightarrow{\mathcal{P}} = \left(\overrightarrow{p} - e\overrightarrow{A} + gv^0\overrightarrow{B} - g\overrightarrow{v}\times\overrightarrow{E} - 2\mathrm{i}f\overrightarrow{E}\right)$.

Making use of the identity (9), we can observe that only the f coupling contributes to the canonical conjugated momentum, given by

$$\overrightarrow{\Pi} = \left(\overrightarrow{p} - \overrightarrow{\mu}\times\overrightarrow{E}\right). \tag{28}$$

As a consequence, it is observed that only the standard Pauli coupling contributes to the A-C phase. The Lorentz breaking non-minimal coupling in the covariant derivative contributes here only with an extra energy term, in the form $4fg\overrightarrow{\sigma}\cdot\left(\left(\overrightarrow{v}\times\overrightarrow{E}\right)\times\overrightarrow{E}\right)$; no phase effect is induced by the Lorentz-violating background vector.

3 Final discussion

In this paper, we have carried out an analysis of the role of possible Lorentz-violating couplings in connection with the Aharonov–Casher phase developed by an electrically neutral particle. Usually, this phase is induced on a neutral particle endowed with a non-trivial magnetic moment interacting with an external electric field generated by an axial charge distribution, but it may also arise in other theoretical contexts. Indeed, it has been here argued that in the case of a non-minimal coupling to the fixed background v^μ (responsible for the Lorentz breaking), an A-C phase is developed even by neutral spinless particles, stemming from the term $g\overrightarrow{v}\times\overrightarrow{E}$ (present in the canonical momentum), where $g\overrightarrow{v}$ plays the role of the intrinsic magnetic moment of the test particle. This is in close analogy to a similar result in $(1+2)$-dimensional electrodynamics: charged scalars, non-minimally coupled to an electromagnetic field, acquire a magnetic dipole moment [32]. In our case, the situation is more drastic: a neutral and spinless particle acquires a magnetic moment, $g\overrightarrow{v}$, as a by-product of the non-minimal Lorentz-violating coupling. Other possibilities have been taken into account as well, such as the non-minimal coupling to the torsion tensor; in this case no A-C phase comes out; instead, we get an extra energy contribution due to the coupling of the spin to the Lorentz-violating background and the electric and magnetic fields. Moreover, for Lorentz violation at the level of the fermionic couplings, parametrized by a skew-symmetric tensor, $T_{\mu\nu}$, it was verified that such a coupling may yield an A-C phase if the magnetic component of $T_{\mu\nu}$, $T^{ij} = \overrightarrow{T}_2$, is non-vanishing. The phase generated here is not obviously shared by scalar particles, as the kind of non-minimal coupling leading to the phase is specific for spin-$\frac{1}{2}$ particles. Actually, the only non-minimal coupling universal for all types of particles, regardless of their spin, is the one given in the covariant derivative according to (2). Finally, a remarkable result is the competition between two non-minimal couplings which separately yield the A-C phase. The case investigated involves the non-minimal standard Pauli coupling and the non-minimal coupling to v^μ analyzed in the first section. Once both interactions are switched on, it is then observed that the A-C phase that survives is the usual one: the one stemming from the term $\overrightarrow{\mu}\times\overrightarrow{E}$, where $\overrightarrow{\mu}$ is the canonical magnetic moment of the spin-$\frac{1}{2}$ particle.

So, as a general outcome, we can state that an interesting effect of breaking Lorentz and CPT symmetries is the possibility to have direct consequences on the A-C phase for test particles, once the latter couple non-minimally to the vector or tensor background that accomplishes the breaking. This is a feature of Lorentz-violating gauge models not yet discussed in the literature. We then argue that in this scenario even neutral scalar particles may acquire a non-trivial A-C phase once acted upon by an external electric field, and we attribute to the Lorentz-violating background vector, non-minimally coupled to the specific test particle, the property of inducing the magnetic dipole moment that couples to the electric field to give rise to the A-C phase. This result is very similar to a mechanism that takes place in planar gauge theories. Indeed, in $(1+2)$D, a number of works [33] have shown how a scalar particle may acquire a non-trivial magnetic moment at the expense of a non-minimal coupling to the Maxwell field. We can actually compare our present result to the $(1+2)$-dimensional counterpart if the violation of the Lorentz symmetry takes place due to an external 4-vector background. The latter sets up, effectively, a $(1+2)$-dimensional world for the interacting particle, and the non-minimal coupling proposed in (2) selects the electric component of the external electromagnetic field, which allows one to identify the combination $-g\overrightarrow{v}$ as playing the role of the spin of the test particle.

To conclude, we would like to mention that, by considering the very general Lagrangian with Lorentz- and CPT-violating terms as proposed by Kostelecký and Lane in [28], it would be worthwhile to select all those specific terms that break CPT and then analyze how they may induce different A-C phases for a particle and its corresponding anti-particle. This study may reveal another interesting and relevant aspect of the standard-model extensions stemming from the breakdown of the Lorentz symmetry. This question is under investigation and we shall be soon be reporting on it elsewhere [34].

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